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LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF A THIN, ELASTIC PLATE OF REGULAR HEXAGONAL SHAPE

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1. INTRODUCTION

The analysis of eigenvalue problems in two- or three-dimensional domains of complicated boundary shape is of basic and applied interest in several fields of science and technology: electromagnetic theory, acoustics, nuclear reactor technology, heat transfer situations, etc. In the case of plates of complicated geometry executing transverse vibrations the problem is of importance in situations as varied as architectural systems, transducer design, electronic pakages, etc. [1]. Several studies which deal with vibrating printed circuit boards have been published [2–5].

The present investigation tackles the problem of a regular hexagonal plate, simply supported and clamped along the boundary, the earliest study being, possibly, the classical treatise by Collatz [6] who determined the lower natural frequency coefficients $\Omega_i = \sqrt{\rho h/D}\,\omega_i L^2$ by means of the finite difference technique. The finite element method is presently applied, making use of a well-known code [7] and the eigenvalues are compared with those available in the technical literature.

2. NUMERICAL INVESTIGATION AND RESULTS

Numerical experiments were run using the net shown in Figure 1 employing rectangular and triangular elements. A total number of 18 004 elements did result; see Figure 1. The same discretization was used for both types of boundary conditions. Table 1 depicts the finite element results obtained in the present investigation for (1) simply supported and (2) clamped edges. The modes have been identified from 1 to 15 according to the modal shapes obtained by means of an elaborate finite element analysis; see Figures 2–11 which correspond to the simply supported situation. The modal shapes corresponding to the clamped plate are similar. The arrangement of Table 1 follows the one used by Collatz in his treatise [6, p. 393] and contains results available in references [6, 4] and those presently obtained. It is important to point out that Collatz analysis was performed for a hexagonal membrane but in view of the analogy prevailing between vibrating membranes and vibrating simply supported plates of rectilinear sides, the square of the eigenvalue of the membrane corresponds to the numerical value of Ω_i of the equivalent thin, elastic plate [8]. The finite difference values selected from reference [6] are the ones which correspond more closely to the results obtained by conformal mapping-variational approach [4] and the ones presently determined since those calculated by Collatz [6] exhibit very large dispersion

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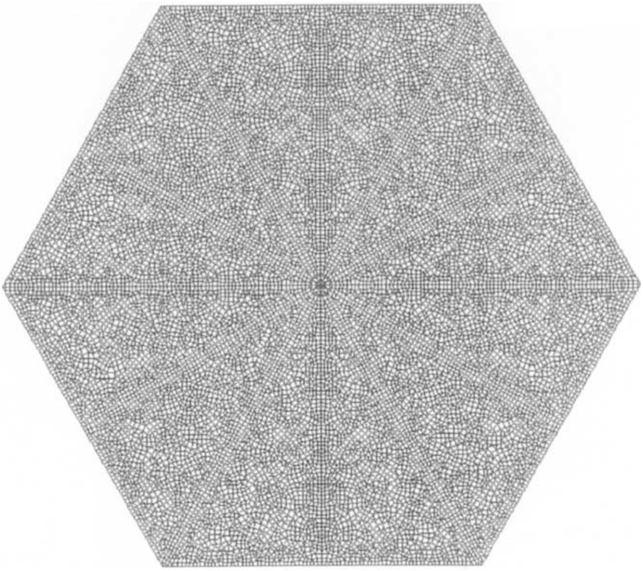


Figure 1. Regular hexagonal plate and finite element net used.

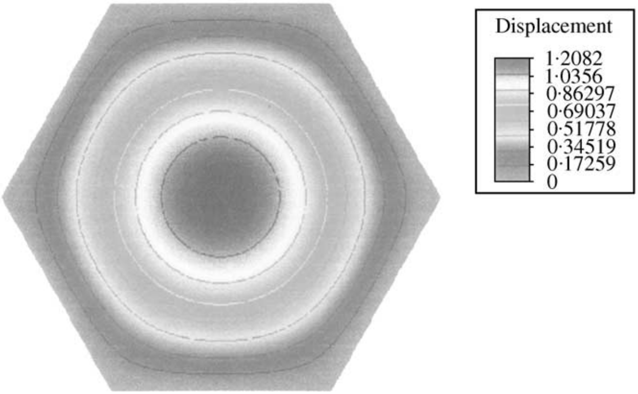


Figure 2. Simply supported hexagonal plate: fundamental mode (Ω_1 = 7·15).

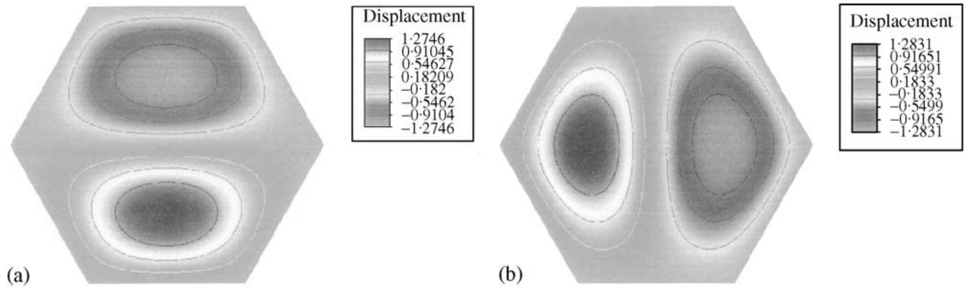
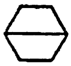
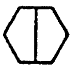
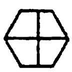
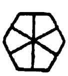
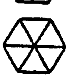


Figure 3. The first two antisymmetric modes ($\Omega_2 = \Omega_3$ = 18·13).

according to the geometry of the cells and their number. For instance, the hexagonal cells did yield the best results but, for the higher modes, the smallest number of cells allowed for the best relative accuracy.

TABLE 1

Comparison of values of frequency coefficients Ω_i of a regular hexagonal plate

		(1) Simply supported				(2) Clamped			
Symmetry about	Nodal lines	[6]	[4]	F.E. Present study	Mode order	[6]	[4]	F.E. Present study	Mode order
G_1, G_2 full symm.		7·030	7·148	7·15	1	10·198	12·851	12·79	1
G_1		16·76		18·13	2			26·52	2
G_2		16·76		18·13	3			26·52	3
G_1, G_2		28·58		32·45	4			43·29	4
—		28·58		32·45	5			43·29	5
G_1, G_2 full symm.		41·0	37·64	37·49	6			49·38	6
G_2		38.2		47·63	7			60·42	7
G_1		36		52·64	8			66·19	8
G_1		42·7		60·11	9			74·90	9
G_2		42·7		60·11	10			74·90	10
G_1, G_2		52·5		70·15	11			85·49	11
				70·15	12			85·49	12
—		52·5		87·54	13			105·15	13
				87·54	14			105·15	14
G_1, G_2 full symm.		90	92·52	90·07	15			108·01	15

The three quasi-axisymmetric modes investigated by means of the conformal mapping approach using three simple polynomial co-ordinate functions investigated almost four decades ago by Laura and co-workers [4] yield eigenvalues which are in excellent agreement with those presently determined for both types of boundary conditions (for the clamped case only the fundamental mode was investigated analytically).

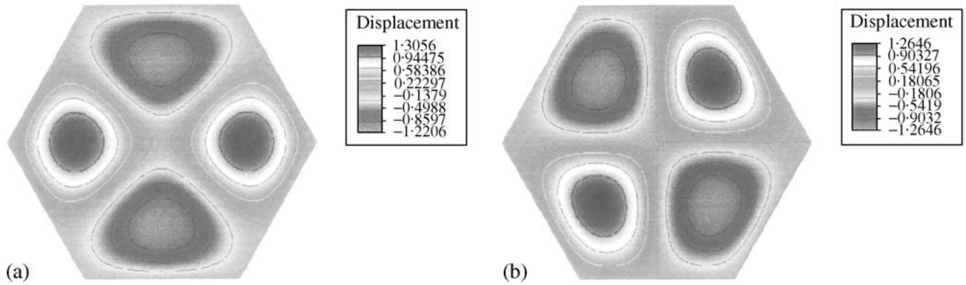


Figure 4. (a) Fourth and (b) fifth modes ($\Omega_4 = \Omega_5 = 32{\cdot}45$).

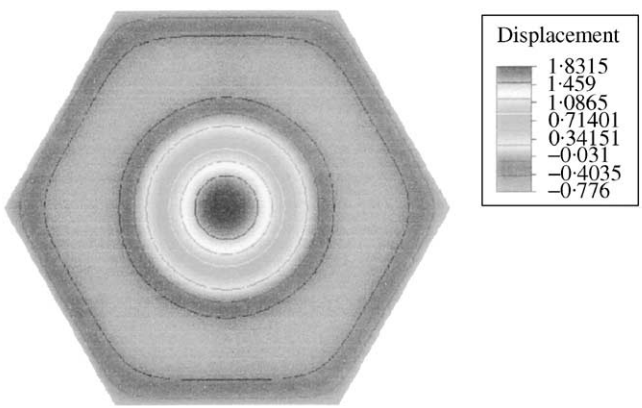


Figure 5. Sixth mode ($\Omega_6 = 37{\cdot}49$): second quasi—axisymmetric mode.

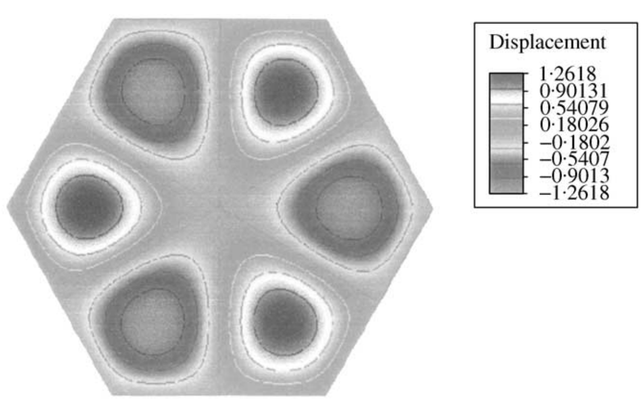


Figure 6. Seventh mode ($\Omega_7 = 47{\cdot}63$).

The fundamental frequency of the simply supported plate determined in reference [6] is in relatively good agreement with the other values shown in Table 1 while in the case of the clamped plate the difference is practically 26%.

Figure 2 depicts the fundamental mode shape ($\Omega_1 = 7{\cdot}15$) while Figure 3 shows the first two antisymmetric modes corresponding to $\Omega_2 = \Omega_3 = 18{\cdot}13$ and Figure 4 shows two different symmetric modes for $\Omega_4 = \Omega_5 = 32{\cdot}45$. Figure 5 depicts the second

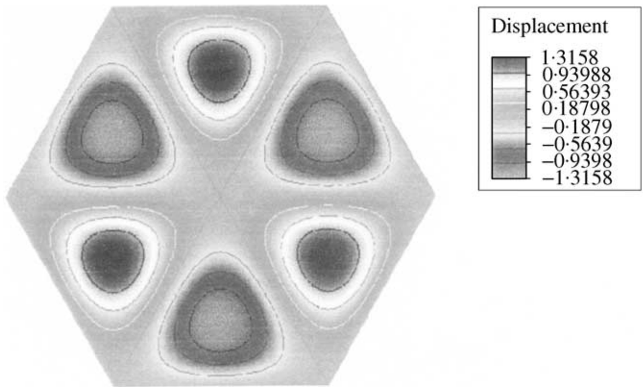


Figure 7. Eighth mode ($\Omega_8 = 52{\cdot}64$).

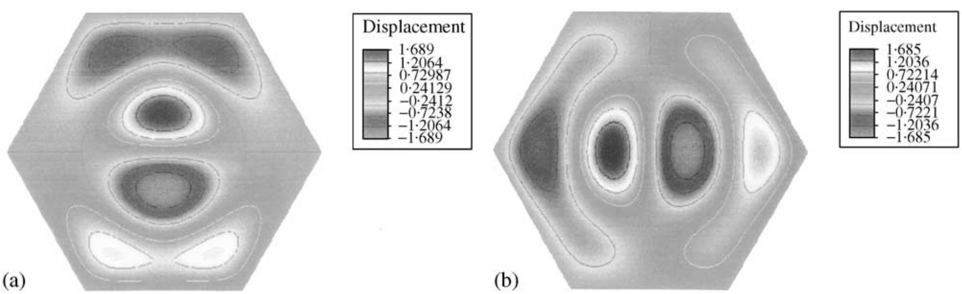


Figure 8. (a) Ninth and (b) tenth modes ($\Omega_9 = \Omega_{10} = 60{\cdot}11$).

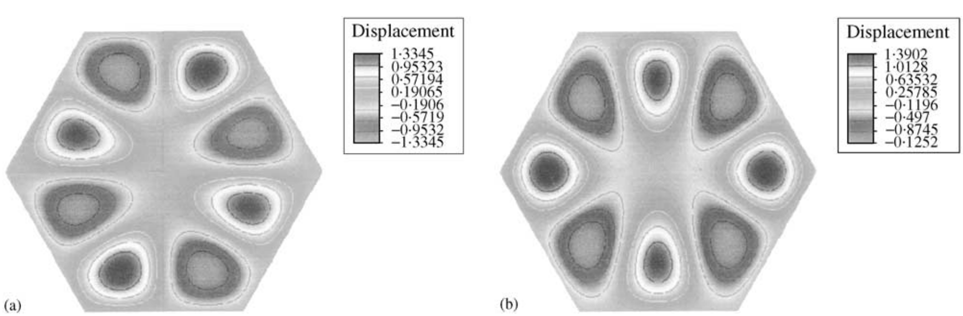


Figure 9. (a) Eleventh and (b) twelfth modes ($\Omega_{11} = \Omega_{12} = 70{\cdot}15$).

quasi-axisymmetric mode ($\Omega_6 = 37{\cdot}49$) and Figure 6 the seventh mode ($\Omega_7 = 47{\cdot}63$) while Figure 7 deals with the eighth mode ($\Omega_8 = 52{\cdot}64$) which is similar to the seventh mode but rotated by $\pi/6$. Figure 8 depicts the rather complicated ninth and 10th modes ($\Omega_9 = \Omega_{10} = 60{\cdot}11$) where a situation similar to the one encountered with modes 6 and 7 is found. The same general geometric characteristic is met with modes 11 and 12 ($\Omega_{11} = \Omega_{12} = 70{\cdot}15$) and modes 13 and 14 ($\Omega_{13} = \Omega_{14} = 87{\cdot}54$), see Figures 9 and 10, correspondingly. Figure 11 deals with the 15th mode (third quasi-axisymmetric modal shape) which takes place at $\Omega_{15} = 90{\cdot}07$. The eigenvalue determined using the conformal

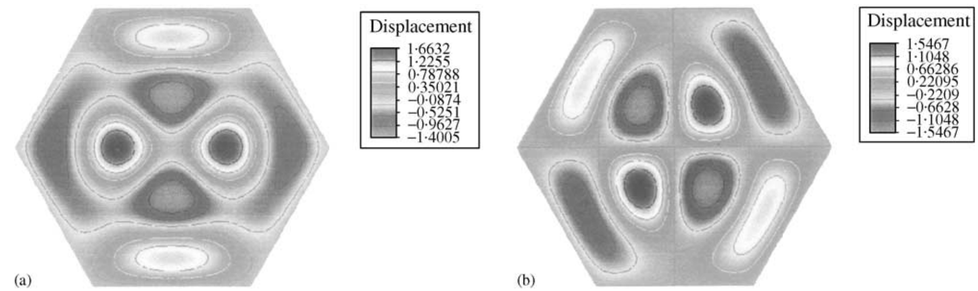


Figure 10. (a) Thirteenth and (b) fourteenth modes ($\Omega_{13} = \Omega_{14} = 87{\cdot}54$).

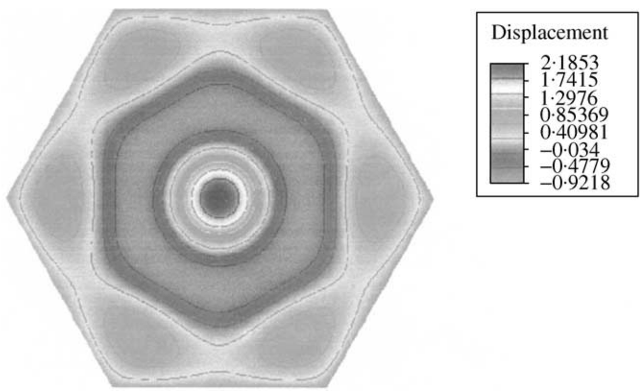


Figure 11. Fifteenth mode ($\Omega_{15} = 90{\cdot}07$).

mapping-variational approach differs from this value by approximately 2% which is certainly remarkable in view of the simplicity of the polynomial co-ordinate functions and considering that only three approximating functions have been employed.

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